Commentary

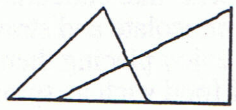
Earth, I

1. **(5 circles should be drawn in the right hand.)**
2. **(4 children)** There are 7 children who like chocolate and 10 who like strawberry. There are 4 children who like both chocolate and strawberry; they are in the overlapping part of the circles. Children might enjoy placing themselves in some loops like this made from rope, for other types of food such as spinach, beans and peas.
3. **(a. 50, 53, 54 ; b. 86, 84, 83; c. 25, 40, 45)** Give a star to a, b, and c separately. Note that (a) is simply counting from 48; (b) involves counting down from 87; (c) is counting by 5's, starting at 15.
4. **(even; odd; even; even; even)** This problem is a concrete introduction to *odd* and *even* numbers. Students might enjoy practicing this process with other numbers of coins.
5. **(12)** The problem introduces students to the *repeating function* concept on a calculator. Most hand-held calculators will repeatedly add, subtract, multiply and divide in this manner. It is interesting for students to experiment with which number that is entered is the one that their calculator repeatedly uses. For the problem $\boxed{5}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$, for example, do they get 17 or 23?
6. **(4)** Many students will intuitively know that half of 8 is 4, so 4 squirrels went to get nuts. Thus 4 squirrels are left behind in the tree. If students have been taught a rule such as "how many are left means to subtract," they might not know how to solve this problem because there is no obvious number to subtract.

Commentary

Earth, II

1. (**5**) There are four small squares and one large square.Students may enjoy doing other problems of this nature, in which they find figures within other figures. For example, how many triangles are in this figure (3):



2. (**9**) 9 + 1; 8 + 2; 7 + 3; 6 + 4; 5 + 5; 4 + 6; 3 + 7; 2 + 8; 1 + 9.
3. (**12¢**) Two nickels and 3 pennies is 13¢, and the difference between 13¢ and a quarter is 12¢. Some students may have trouble with this problem if they don't know the value of the coins.
4. (◆) The pattern which repeats is ■◆◆♥♥♥. The fourth repetition of this pattern has started, and the first two figures are shown, leading to the third in the sequence as the one to follow.
5. (**a. 559; b. 850; c. 1,272**) Give a star for a, b, and c separately.
6. (**One possible answer is shown; there are other possible answers.**) Students may enjoy knowing that this is related to one of the "50 famous unsolved problems in mathematics" of the 80's. The problem was that everyone thought that such a map could be colored in four colors or less, so that no two boundaries the same color touched except at a point, but no one could prove it. Eventually the problem was solved, but for years and years, mathematicians enjoyed coloring maps like this, looking for an exception to the conjecture.

Commentary

Earth, III

1. **(4)** The bag needs to have four apples in it so that the scales will have the same weight on both sides. This assumes that all apples weigh the same. This problem is an important one to lay a concrete foundation for algebraic thinking.

2. **(8)** The student may think *what number, plus 9, equals 17.* Eight + 9 = 17 is part of a family of facts which also include: 9+ 8 = 17, 17 - 9 = 8, and 17 - 8 = 9.

3. **(a. 3 ; b. Pirates; c. 3)** The Hornets won 5 games; the Pirates won 4 games; the Eagles won 2 games; and the Bears won 1 game. For part a, the Hornets won 5 games and the Eagles 2 games, which is 3 games more. For part b, the Pirates won two more games than the Eagles. For part c, the student might want to get 12 pennies and move them around until he or she gets the same number in 4 different piles. If the student "even outs" 12 into 4 piles, he or she will get 3 wins; or said a different way: $12 \div 4 = 3$. This is a concrete introduction to the concept of getting an *average*.

4. **(a. 44; b. 32; c. 52)** The student may use "guess- check-revise" to find the answer by repeatedly trying different numbers for each box until they get one which works. Some students might realize that they can solve a different problem than the one given. For (a), they might solve by adding: $23 + \square = 67$; or they might solve by subtracting: $67 - 23 = \square$. Problems (b) and (c) can also be worked by solving a different problem.

5. **($1.28)** The student subtracts the value of the coupon, 25¢, from the cost of the apple butter, $1.53, giving $1.28.

6. **(4)** Purchasing 3 boxes of markers would provide 27 markers since 9 + 9 + 9 = 27. One more box is needed to give one marker per student, but 7 markers would be left over.

7. **(24)** The two insects would have 6 + 6 or 12 legs to offer to the collection. The three frogs would have 4 + 4 + 4 or 12 legs to add also. Therefore there's a total of 24 legs. This is a multistep problem which students can solve by drawing a picture of the frogs and insects, and counting legs. Or they might use the picture given in the problem, and count the legs that way.

Commentary

Earth, V

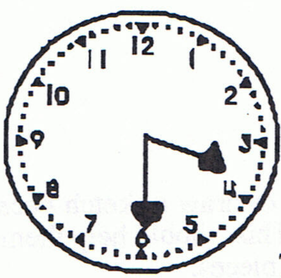
1. **(Thursday)** The students may make a calendar, starting with Wednesday the 8th, and putting the numbers in from 7 to 1, then from 9 to 16.

2. **(tape holder)** This problem will be difficult to many students who do not have an intuitive understanding of balance situations. It will be difficult for them to see that if 3 of object A weighs the same as 2 of object B, then B must be heavier. Actual balance scales in the classroom would help to see this inverse relationship between the number of objects to be a certain weight, and the weight of an individual object.

3. **(First: car; Second: van; Third: truck)** Students might act it out or find it helpful to write each word on an index card and move the cards around until each vehicle is in the correct order.

4. **(a. 23; b. 12; c. 38)** The student may use "guess-check-revise" to find the answers: 46 - 23 = 23; 30 - 18 = 12; and 24 + 14 = 38. They also might do different problems from the ones given, by solving a related problem such as 46 - 23 = $\boxed{}$ or 23 + $\boxed{}$ = 46 for (a), and so on for (b) and (c).

5.



The student should understand the hour and minute hands on a clock. The hour (shorter) hand should be between 3 and 4; the minute (longer) hand should be on the 6.

6. **(a. 18; b. 8; c. 8)** This problem is related to Venn diagrams, which students have likely met in first grade. They may need to be reminded that numbers can be in more than one figure. For part a, the numbers in the rectangle are 8, 9, and 1: 8 + 9 + 1 = 18. For part b, the numbers in the rectangle and circle are 2 and 6: 2 + 6 = 8. For part c, the numbers in the rectangle and not in the circle are 5 and 3; 5 + 3 = 8.

7. **(40¢)** This problem can be solved in 2 steps, by adding the two numbers and subtracting their sum from 79¢, or by subtracting one number from 79¢ and then the next number from what is left. In either case, the answer is 40¢.

8. **(about $5)** Students should realize that 95¢ is close to $1, and that there are 5 school days in a week. Therefore it will cost about $1 a day for five days, or about $5 for lunch at the school for a week.

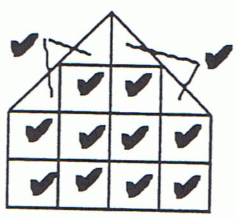
Commentary

Earth, VI

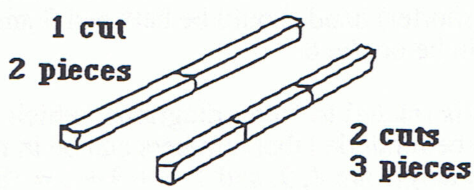
1. **(15)** Students might first add 9 and 12 and then subtract 6, or they might realize that only half a dozen, 6, need to be added to 9. Some students might not know what a dozen means, but having the egg carton shown should be a hint. Most students will intuitively know what "half" means in this situation, and can count half the eggs shown for "half a dozen."

2. **(a. L, N; b.  ,  ; c. 54, 49)** In pattern a, the pattern skips one letter each time. In pattern b, the dog, pencil, dog, cake pattern repeats. In pattern c, 5 is subtracted from the previous number each time. In the last pattern, some students might get the answer by the rhythmic count of numbers that end in 9 followed by numbers that end in 5, working backward through the decades.

3. **(12)** The concept of area in this problem includes "half-squares." It is helpful for students to use figures where the halves fit together to make another whole unit square rather than counting "half" each time. In the figure given, each ✔ is one whole unit square.



4. **(20 minutes)** It might be helpful for students to act it out, or draw a sketch because some might think that two pieces would require two cuts. This should help them see that only two cuts are required, at ten minutes each, to get three pieces.



5. **(Annie: 25; Baldwin: 34; Carl: 18)** Students may use "guess-check-revise" or logical reasoning to solve this problem. If the boys have *even* numbers on their shirts, Annie must have the only *odd* numbered shirt. Baldwin's number must be even and have a sum of seven; the only number with these characteristics is 34. Carl's number then must be 18.

6. **(clown: (5,2); train: (2,1); elephant: (3,4))** It is important for students to realize to go east (right) first, then go north (up) to locate points. For students having trouble, have them trace the path with their finger.

Commentary

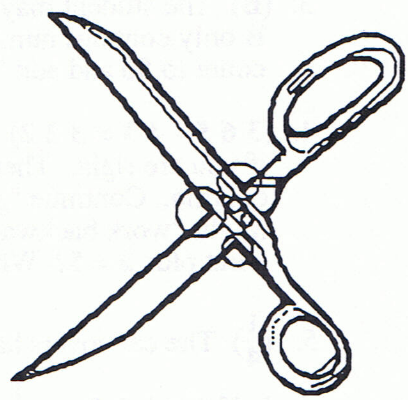
Earth, VII

1. **(90¢)** Students may identify the pattern as "adding on" 15¢ each time.

2. **(a. TV; b. sleeping; c. eating)** Students have an opportunity to work with a circle graph to answer each question. The answers are based on visual estimates of the size of one region as compared with another.

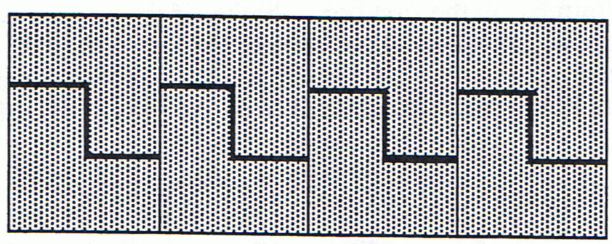
3. (>) 28 > 27

4. **(See below.)** The drawing to the right has several angles circled. Be a little generous with checking the paper. For example, if students circle a sharp point of the scissors, give them credit although technically part of the tip has a curved edge.



5. **(7)** Starting with the first clue and proceeding in order, the only numbers whose sum is 3 are 1 and 2, so mark them out. The only 2 numbers left whose sum is 8 are 3 and 5, so mark them out. The only 2 numbers left whose sum is 12 are 8 and 4, so mark them out. The only 2 numbers whose sum is 15 are 6 and 9, so mark them out. Seven is left

6. **(8)** Students might be encouraged to cut out a shape like the one shown, and physically move it around the grid to cover it. Such an arrangement is shown below.



7. **(5 centimeters)** Students might take a piece of string and curve it to fit the mouse's tail, and then measure the string. Or they might try measuring with a straight-edge centimeter ruler -- if so, they might have selected 2 as the estimate unless they somehow "go around the curve" in small chunks. 10 and 13 centimeters should be obviously wrong.

Commentary

Earth, VIII

1. **(3)** January 21st is a Monday. Three Sundays have already passed in January: January 6, January 13, and January 20. The student can locate January 21, move backward a space to the Sunday column, and count backwards three Sundays in that month.

2. **(28)** The student must know what "triangle" means, and also know that there are "overlapping" triangles in the drawing. There are 8 triangles in the cat's head --each eye contains 3; 13 triangles in the cat's body, and 7 triangles in the cat's tail: 8 + 13 + 7 = 28 in the entire body.

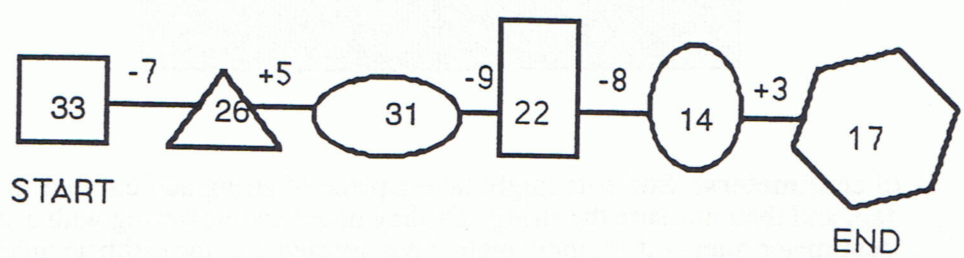
3. **(B)** The student may look for a pattern in several ways. The student may observe that column B only contains numbers that end in a 7 or a 2. Or a student may look at column E, mentally count to 50 and add "2 more." Or the student may complete the chart to make a list.

4. **(3 6 5 - 4 3 = 3 2 2)** Start in the ones column. "Guess" a number and then "check" to see if you are right. Then go to the tens column and "guess and check." End in the hundreds column. Continue "guessing and checking" until you find the right number. Or the student might "work backwards" by turning the subtraction situation into an addition one; for example, what plus 2 = 5? What goes in the box must be 3. Continue in this fashion.

5. **($\frac{1}{4}$)** The car covers half of the circle; the robot and the telephone each cover $\frac{1}{2}$ of the half that is left, or $\frac{1}{4}$. The chance of landing on the telephone would be "1 out of 4," or written as a fraction: $\frac{1}{4}$

6. **(50 feet)** It might be helpful to draw a picture. By drawing one "pole" or "dot" and then continuing until a total of 6 are drawn, one can understand that there are five spaces between the six poles. Each space is 10 feet, so 10 + 10 + 10 + 10 + 10 = 50 feet in all.

7. **(33, 26, 31, 22, 14 go in the shapes.)** The problem can be solved in several ways In "guess-check-revise," try a number in the first box and calculate across; if the ending number is not correct try another number in the first box -- higher if the answer was too low, and lower if the answer was too high. Continue until the correct number is found. Or work backwards, by starting with the number you know, 17, and asking what number, when added to 3, gives 17? The number is 14. Continue working backward from the right end to the left end, in this fashion.



Commentary

Earth, IX

1. **(6 hours)** It is helpful to draw a picture of the lizard's trip. At hour 1, the lizard started at 0, went up to 2, and down to 1. At hour 2, the lizard started at 1, went up to 3, and down to 2. At hour 3, the lizard started at 2, went up to 4, and down to 3. At hour 4, the lizard started at 3, went up to 5, and down to 4. At hour 5, the lizard started at 4, went up to 6, and down to 5. At hour 6, the lizard started at 5, went up to 7, and climbed out!

2. **(4 hours)** The essence of this problem is to know that Howard watches T.V. from 11:15 to 12:15, from 12:30 to 1:30, and from 5:30 to 7:30. The first and second times he watched for an hour each, and the third time for 2 hours, totalling 4 hours in all.

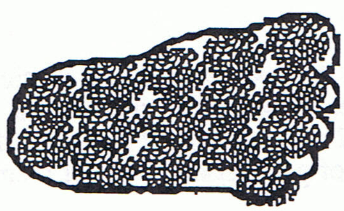
3. **($10)** A student can use a calculator, but many won't need one. Intuitively they can add $2.50 to itself to get $5, twice, and $5 + $5 is $10. It would be interesting to see the other strategies that students use on this problem.

4. **(b. less than 50 grams)** If the hot dog and bun were exactly 50 grams, the scale would be even. Since the 50 gram weight is lower it must be heavier. Therefore, the hot dog and bun must be less than 50 grams.

5.

The pattern repeats after every four figures. The 15th figure, then, will be identical to the 3rd figure. Some students will recognize this, but some may need to draw each figure out to the 15th.

6. **(accept between 13 and 21 as an answer.)** The figure below shows 15 thumb prints, which cover the footprint but with some "holes." The problem should encourage estimation since an exact answer can't be obtained by the students.



7. **(16)** Since students do not know how to divide yet, they will try a number of different strategies to find the answer. One is to ask yourself "what number taken twice will give a sum of 32?" Students might try a few numbers and see.

Commentary

Earth, X

1. **(1979)** Subtract 16 from 1995 and get 1979. Or, a student might be successful by counting backwards 16 times from 1995.

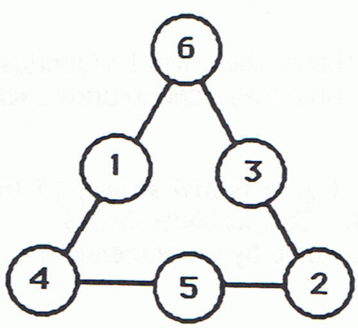
2. **(6,528)** This problem can be solved using the "guess-check-revise" method, using the numbers given: 2, 5, 8, and 6. Students might put these four numbers on index cards, and physically move them around until they find the right combination. The number has to begin with 6, so that card would stay stationary while the student moves the other three.

3. **(a. 7; b. 15)** In part (a), note that the graph is shaded halfway between 6 and 8, so there are 7 that like raspberry. In part (b), 5 second graders like lemon and 10 like strawberry for a total of 15. Students who have not seen graphs in which all the numbers are shown on the bottom axis might have difficulty with the problem for that reason.

4. **(9 and 12)** There are several ways to approach this problem. One way is to use "guess and check" until you have the correct pair of numbers. Another way is to make a list of pairs of numbers that equal 21:

   1 + 20 4 + 17 7 + 14 10 + 11
   2 + 19 5 + 16 8 + 13
   3 + 18 6 + 15 9 + 12 -- only this pair differs by 3

5. **(see below)** Some students may use " guess and check" until they find the right combinations for 11. Note that the three sides of the triangle can be switched around.



6. **(c should be circled)** Notice that the last figure has 4 vertices (points where the paths meet), and each has an even number of paths coming out of it. Networks such as these are *traceable* if they have exactly 0 or 2 odd vertices. This network has 0 odd vertices, since all four vertices have an even number of paths coming out of them.

7. **(6)** Students can see that a stapler weighs 12 grams from the smaller scale. Therefore on the larger scale, the two staplers weigh 24 grams. Since the entire weight is 30 grams on the big scale, the ball must weigh the difference between 30 and 24, which is 6.

8. **(15 + 8 = 23 or 18 + 5 = 23)** Students can again take five index cards, but this time label them =, 5, 1, 8, and +, and arrange them to give 23 as an answer.

Commentary

Earth, XI

1. **(about 30)** Subtracting "forty something" from "70 something" might give about 30. It couldn't give a number close to 50, as 79 – 40 would give the highest difference, 39. Likewise, "about 110" is unreasonable, although some students might get it because they add instead of subtract.

2. **(Yes)** The cost of the two items is 79¢. Maria has 95¢. She has enough money to buy both. Students might want to count her money using real coins, or use a calculator.

3. **($1.75)** Students need to know that a week has seven days. Some students might know the answer is 7 quarters, but not know how to convert that amount into dollars and cents. Give them 1 star for such an answer.

4. **(800)** Students should use their intuition that 288 is close to 300, and 497 is close to 500, and 300 plus 500 is 800.

5. **(<u>65</u>, <u>67</u>, 69, 71, 73, 75, <u>77</u>)** Students should see the pattern of counting by 2.

6. **(1)** Students should draw 6 rocks in the right hand, giving a total of nine. This is one less than 10 fingers.

7. **(a. Lee; b. Sept. 21; c. John; d. 5)** Students who are familiar with a calendar should have no difficulty with this problem.

8. **(12)** Students may solve this by *working backwards* or by *guess-check-revise*. To work backwards, they start at the end number, 10, and ask themselves what the previous number would have to be so that, when 5 is subtracted, 10 is left. They would get 15 as the next-to-last number. Then they would work backward again by asking what number, when 3 is added, gives 15. That number is 12, which is the starting number. To guess-check-revise, students would simply guess a start number and do the arithmetic. If that wasn't correct, they would guess a different start number, either higher or lower than the first, based on what happened with the first.

Commentary

Earth, XII

1. **(8)** Students may use several strategies to solve this problem. They might total all the animals found, then subtract all that escaped. Or they might subtract the number of each type that escaped from the total of that type, and add the remaining animals. Drawing a picture would help, and then the answer can be found by counting.

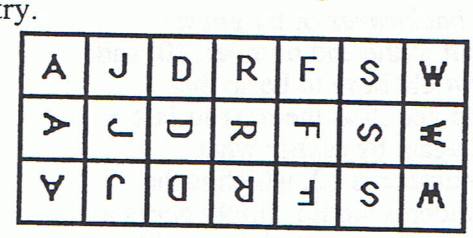
2. **(84)** There are two ways to solve this problem. Under *guess-check-revise*, you would "guess" a number and check to see if it is right. If not, revise your guess until the solution is found. For *working backwards*, the student starts with the answer 83 and asks "what was the previous number so that, after 7 is subtracted, 83 is left?" The number is 90. Then work backwards on the previous step asking, "what number did I start with so that, after adding 6, I got 90?" The number is 84. Still a third way to approach the problem is to notice that 6 is added and 7 is subtracted in the middle of the pond, meaning a total of 1 is subtracted. So the problem becomes, "what number do I start with, so that when 1 is subtracted, 83 is left?"

3. **(12)** Students can be encouraged to solve this problem by "making an organized list -- 26, 27, 28, 62, 67, 68, 72, 76, 78, 82, 86, 87. Notice the list starts with the smallest number, a 26, and then list all the others that start with 2 in the tens place. Then the list moves to the next largest number in the tens place, and so on.

4. **(first circle: $\frac{1}{4}$; second: $\frac{1}{3}$; third: $\frac{1}{2}$)**

   Through observation or using concrete examples, students should realize that there is one out of four equal parts shaded in the first circle; there is one out of three equal parts shaded in the second circle; and there is one out of two equal parts shaded in the last circle.

5. **(see below)** Visual discrimination is involved in solving this problem. Each letter in the top row is turned 90 degrees to get the letter below it, and another 90 degrees to get the third entry.



6. **(40; even)** Students can learn to count such collections by "counting by twos." If they do so, the collection is *even* if they can count the whole set and end on one of their counting by twos numbers. The collection is *odd* if they have one left over, counting by twos.

7. **(120 minutes)** 60 minutes in an hour + 60 minutes in an hour = 120 minutes in 2 hours.

8. **(see below)** Students have a chance to make their own pictograph in this problem. They will have to think of the money earned as dimes (for example 40¢ is 4 dimes).

Marsha: OOOO
Danny: OOOOO
Molly: OOOO
Bruce: OO

Commentary

Earth, XIII

1. **(1/2 or 2/4)** 2 out of 4 equal-size parts or 1/2 of the circle is stripes. Students are equating the area of parts of a figure with the probability of landing on that area.

2. **(218)** The *even* numbers are: 14, 88, 100, and 16. Students might want to remember that the even numbers are the ones you would say aloud if you counted by twos. They could count by twos, from 2 to 100, and check off each of the numbers given if they called out its name.

3. **(The missing digits in order from hundreds to ones are: a. 3, 5, and 1; b. 5, 4, and 5)** Students may solve these problems by turning each box in a column into a missing addend problem.

$$\begin{array}{r} \square\square\square \\ +\ \underline{3\ 0\ 2} \\ 6\ 5\ 3 \end{array} \qquad \begin{array}{r} \square\square\square \\ +\ \underline{2\ 2\ 3} \\ 7\ 6\ 8 \end{array}$$

4. **(James)** Students may "act out" the problem to help solve it, or draw a diagram or make a list. To make a list, they would put Carolla on top of Tremaine to indicate Carolla is older, and then put James above Carolla for the same reason. Then James would be on top, Carolla next, and Tremaine last, indicating their age.

5. **(a. 5; b. 8; c. 15; d. 28)** The line plot may be new to students, but the key should help them realize it is somewhat like a pictogram. Two students drink no milk, five drink 1 glass, four drink 2 glasses, and five drink 3 glasses each day.

6. **(6)** Students may see this pattern in a real-world situation:
   1 gallon requires 2 ounces,
   2 gallons require 4 ounces,
   3 gallons require 6 ounces.

   This is an introduction to ratio, but at this stage can be thought of as a pattern problem, a repeated addition problem, or simply a counting problem.

Commentary

Earth, XIV

1. **(8)** If one cube weighs 4 grams, 2 cubes weigh 8 grams. The scale reads 16 grams, so the can (cylinder) must weigh 8 grams.

2. **(a. 5; b. 18; c. 49)** Students may find the missing factor by asking themselves "what number could the box be covering so that the sentence is true. They would try different numbers and check to see if they are correct. Some students might turn the problem into a different but related problem, addition to subtraction or vice-versa. For (a), they might find 11 – 6; for (b), 28 – 10; for (c), 44 + 5.

3. **(C. 4 inches)** This problem might be solved with only visual estimation skills, but may also be solved by physical means. A student might spread their fingers apart the same distance as the pencil is long, and then see that they can put their outstretched fingers about two times across the sheet of paper. Or, they might mark the pencil's length on another sheet of paper, and move the marks in the same manner as with their fingers. The pencil is about one-half the width of the paper.

4. **($13)** I L O V E M A T H
   $1 + $2 + $1 + $2 + $1 + $1 + $1 + $2 + $2 = $13

5. **(5:00 p.m.)** Students need to know how to read and write time to the half hour, and that 1/2 hour is 30 minutes. They might proceed by adding the 2 1/2 hours in "chunks." For example, they might start at 2:30, and add 1 hour to get 3:30, then another hour to get 4:30, then the last half hour to get 5:00.

6. **(a. 5; b. 7; c. 5)** Part (a) involves reading the chart correctly, then subtracting Lisa's 2 points from John's 7. Part (b) involves adding the player's scores for each team, and then subtracting 12 from 19. Part (c) involves thinking about the second turn, and subtracting from that total what Suki had on the first turn.

7. **(14 cubes)** Again, visual skills are necessary for this problem. Students should realize that they are looking at a 3-dimensional picture. There are 9 cubes on the base, 4 in the middle, and one at the top.

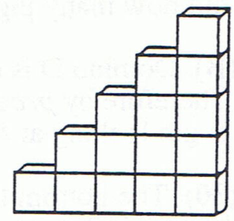
8. **(SELL)** 11004 – 3269 = 7735, which, turned upside down, spells "SELL".

Commentary

Earth, XV

1. **(15)** The problem is one with extraneous information. Many students want to do something -- add or subtract -- the numbers 27 and 18, because those are easily recognized. These students might be encouraged to draw a diagram of the kids and the adults, including 12 kids in the egg-toss contest.

2. **(15)** Students can either make the next two sets of steps in the pattern to get one five steps high, or they can draw it and count. Such a set is shown to the right.



3. **(a. Thursday; b. Friday; c. Tuesday and Wednesday)** For part a, 21 is the highest number, so the day must be Thursday. For part b, find "11" on the chart, look across and see Friday. For part c, 15 + 10 = 25, so Tuesday and Wednesday are correct.

4. **(◆)** Let *p* stand for plane, *h* for heart, and *d* for diamond. The pattern repeats every six times: *p, h, d, p, d, h,* etc. Some students will think of the pattern as one which repeats after three figures, with the 2nd and 3rd figures (heart, diamond) alternating which comes first.

5.

Pennies	Nickels	Dimes
15	0	0
10	1	0
5	2	0
5	0	1
0	3	0
0	1	1

These can be in any order. Encourage students to make combinations of real dimes, nickels, and pennies to fill in the chart.

6. **(The circle is cut into thirds.)** Students may see the pattern as the first circle cut into sixths, the second circle cut into fifths, the third circle cut into fourths, and the next circle cut into thirds.

7. **(b)** It is more likely to land *down* than *up*. The thumbtack landed *down* 68 times; it landed *up* only 32 times. It seems likely from this experiment to land *down* about twice as often as *up*.

8. **(see below)**

6	+4	10	-3	7	+4	11	+8	19

Commentary

Earth, XVI

1. **(4 dimes and 4 nickels)** Students might just start by *guess and check*. Or they might realize that 1 dime and 1 nickel is 15¢, then double that and get 2 dimes and 2 nickels at 30¢, double that and have 4 dimes and 4 nickels at 60¢.

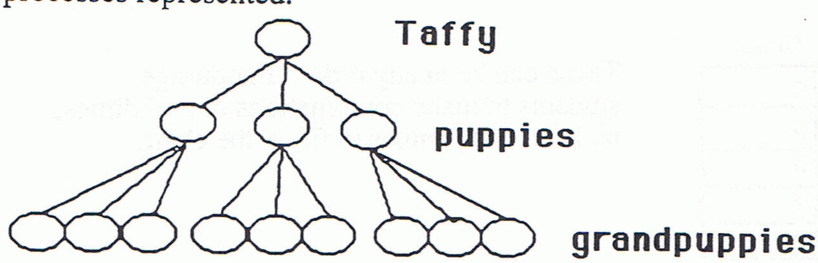
2. **(26)** Subtract the 21 pigs who did not finish the race from the total number of pigs, 47, to find out how many pigs did not finish the race.

3. **(B)** Domino D is eliminated by the first clue, C by the second clue, and A by the third. Therefore by *process of elimination*, B is the only one that fits all the clues. Students should begin looking at such problems in the future as being solved by *process of elimination*.

4. **(30)** The bottom layer of the steps is 4 cubes by 3 cubes or 12 cubes. The layer next to the bottom is 3 cubes by 3 cubes or 9 cubes. The layer next to the top is 2 cubes by 3 cubes or 6 cubes. The top layer is 3 cubes. $3 + 6 + 9 + 12 = 30$. Students might be encouraged to actually build such a set of steps using Unifix cubes or sugar cubes.

5. **(Rose, Sue, George)** The first clock shows 4:30 which is a reasonable time for soccer practice, where Rose was headed. The second clock shows 3:00 which is time school might get out, as Sue mentions. The third clock has 8:00, which is a reasonable time for school to begin.

6. **(9)** Drawing a diagram will help solve this problem. A drawing such as the one below will help. Notice that the drawing doesn't try to show a dog itself, but rather has the thought processes represented.



Commentary

Earth, XVII

1. (**8 thousands**) Eighty hundreds is 80 starting in the hundreds place or 8,000.

2. **(138, 204, 73, 160)** Students will likely have no trouble with the top two problems, but some might have trouble with the last two since they involve subtraction. They should be encouraged to check their work by rounding the numbers and mentally doing the problem with easy numbers, to see if the calculator answer is close enough to assume they didn't make a mistake.

3. **(10 eggs)** The problem involves two steps, and knowing that a dozen eggs is 12 eggs. Students might add 4 to 12 and then subtract 6, or they might draw a picture of the eggs and simply count.

4. **(40^{o} C; 70^{o} C; 55^{o} C)** Each line on the thermometer equals 5 degrees. The longer lines are multiples of 10. The shorter lines are multiples of 5. Students may have trouble reading between the marked lines for the third temperature.

5. **(30 servings)** 5 containers of dog food times 6 servings per container equals 30 servings. Students may also solve this by drawing a picture of each container with 5 servings in each. This may also be solved with repeated addition. $6 + 6 + 6 + 6 + 6 = 30$

6. **(July)** 187 rounds to 200. 198 rounds to 200. 211 rounds to 200. All scores during the month of July will round to 200.

7. **(15)** When you count by 3's, you get 3, 6, 9, 12, 15, and 18. When you count by 5's, you get 5, 10, and 15. The number less than 20 that is found in both is 15.

8. **($4 + 4 = 8$; $2 + 2 = 4$; $3 + 3 = 6$)** The answers given are the most common ones, but students might arrive at different answers by looking at the pictures differently. In the first picture, they might see 1 hand plus 1 hand, rather than 4 fingers plus 4 fingers. In the second picture, they may see 1 plate plus 1 plate, rather than 2 eggs plus 2 eggs.

Commentary

Earth, XVIII

1. **(from the left edge, Clint, Sandra, Billy, Margie, and Freida)** The given picture is harder to use than simply making your own line from the letters of the alphabet standing for the swans' names because, from the first clue, you don't know which swan is Freida -- all you know is the relative position.

2. **(70)** This problem has information in it that is not necessary to solve the problem. Add the 50 miles to the 20 miles to get the answer of 70.

3. **(a. 2 b. 3)** Students will use different approaches to estimating this height. Some may do so visually, although it's somewhat difficult since the pencil is horizontal and the stick figure heights are vertical. Another method would be to find an object as long as the pencil, and use that object repeatedly to estimate the height. Still another method would be to mark off the distance of the pencil on a piece of paper, and use it repeatedly to gain an answer.

4. **(24¢)** Subtract a dime (10¢) from 34¢.

5.

26	*15*	11
17	12	*5*
9	*3*	*6*

6. **(19)** Students might draw a picture to illustrate the problem. The last two inches are divided by one snip. Another approach is to begin with an easier length and find the pattern. If the ribbon is 3 inches long, two snips will cut it into one-inch pieces. Studetns will discover that the number of snips is always one less than the lenght of the ribbon.

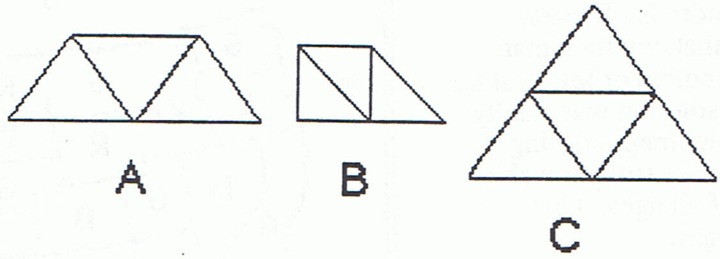
7. **(45)** From the first two sentences, students know that there are either 85 or 45 beans. From the third and fourth sentences, there must be either 45 or 25 beans. The number in common to both possibilities is 45.

8. **(250)** Students will solve this problem in a number of ways. Some might draw all 25 cookies and the chips in each, and simply count. Others will find an easier way, such as drawing 25 cookies and counting by tens. Others will try various ways of grouping the cookies -- for example, 10 cookies would have 100 chips, so they might group by 10 cookies, 10 cookies, and another 5 cookies.

Commentary

Earth, XIX

1. Several of the correct ways to divide the shapes are as follows:



2. **(A. 40 B. 40)** Problem (a) involves the repeated function concept on a calculator. Four is added repeatedly, every time $\boxed{=}$ is pushed. Problem (b) is used to show that multiplying by 10 is the same as adding a number (the number 4 from part (a)) ten times.

3. **(Princess Dianne , 2 more buttons)** Dianne had 20 + 2 = 22 buttons. Joy had 5 robes with 4 buttons each, which is 20 buttons. Students can find these numbers by drawing the figures and simply counting, if necessary.

4. **(12:30 p.m.)** 11:00 plus one and one-half hour gives 12:30 p.m.

5. **(12)** Students might want to draw a diagram to decide what a reasonable answer might be. Without such an aid, many will think that the numbers should be added. They might not understand what such a road sign means.

6. **(Ann)** A costs \$3. Each N costs \$5. \$5 + \$5 + \$3 = \$13. Students might enjoy finding out who has the most expensive and least expensive name in the class, using these charts.

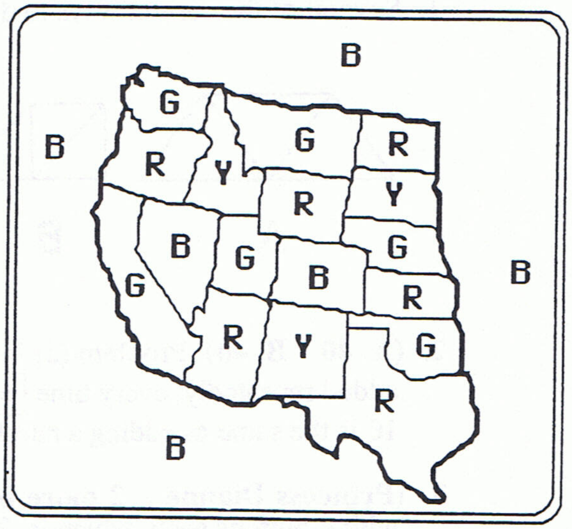
7. **(1)** Sunday the 27th is the last Sunday left in the year. Students will need to know that there are 31 days in December. They might begin by writing the days of the week with numbers under them, calendar-style, until they run out of days in December.

8. **(A. 376 B. 504 C. 265)** These problems are not difficult, except that the order in which they typically appear in textbooks – largest to smallest – has been scrambled. Therefore the student must first decide the order to put the numbers in, so the place value becomes obvious. Problem (b) might give difficulty since there are no 10s to consider, and some will not remember to record a 0 in the tens place of the answer.

Commentary

Earth, XX.

1. **(To the right.)** This problem is the famous "four-color problem" from the ancient history of mathematics. For hundreds of years, mathematicians thought that any such map could be colored in four colors or less, but no one could prove it. The solution was finally reached in the mid-80s, but map-coloring exercises such as this one are still enjoyable for students and adults of all ages. One solution is given to the right:



2. **(>)** The number sentence is "38 is greater than 35."

3. **(a. 6; b. 2; c. 5)** Students might put the digits from 1 to 9 on index cards, and first try each riddle with a card pulled at random, then move to a higher or lower digit from the index-card pile if that guess didn't work. This would be a concrete introduction to the *guess-check-revise* strategy.

4. **(30)** The answer may be obtained by adding 5 six times. Students may want to draw a picture of the 6 flower pots, with 5 flowers in each, and simply count.

5. **(167, 289, 305, 430, 521)** This answer is found by place value. Since each number has a different value in the largest place, the hundreds, students only need to look at the hundreds place.

6. **(382, 328, 832, 823, 238, 283)** The numbers may be listed in any order. However, students should be encouraged to *organize* their work in such cases. For example, this list is organized by "make all the numbers you can with 3 as the first digit, then move to 8 as the first digit, then to 2 as the first digit, as the digits appeared in the problem."

7. **(11)** To solve this problem, all you have to do is add 3 years to Ronnie's age to get Chauncey's age, 9. Then add 2 years to Chauncey's age to get the age of Quartasha. Some students might want to simply hold out 6 fingers for Ronnie, add 3 more fingers for Chauncey, and then 2 for Quartasha, and count.

8. **(Mae)** The first clue eliminates Sabrina. The second clue eliminates Jenny. The third clue eliminates Dee. Therefore by *process of elimination*, Mae is the answer. Notice that Mae fulfills all three conditions.

Commentary

Earth, XXI

1. **(201 people)** Some students might not realize that a plane from Atlanta to Miami might make an intermediate stop in Orlando. A diagram might be helpful. The problem can be approached by either adding 186 and 20, then subtracting 5, or by subtracting 5 from 20, and adding that result to 186.

2. **(dumptruck)** The total of the money is $1.45 which is the cost of the dumptruck. The roadgrader is also less than $1.45 but it is not the most expensive.

3. **(yellow, blue, red, purple, green, from left to right)** The second clue (the yellow car last) gives you a place to start. Label the left-most car (the last car in line) as yellow. The first clue then tells you then blue is next-to-last, and red is in the middle of the 5. The third clue tells you that the green car must be first, followed by the purple.

4. **(9 cookies)** The most common way to solve this problem is to add all the cookies known to be eaten, then subtract from 25 to find what Dad ate. Students might also draw 25 cookies, mark out those they know were eaten, and count the ones left.

5. **(a. 2; b. 6; c. 9; d. 2 and 7)** These problems can be solved by working backwards from what you know. Parts (c) and (d) are more difficult as they involve regrouping.

6. **(30 squares)** Students should be encouraged to organize their search for these squares. Perhaps the easiest way to count all the small squares first, then move to the next smallest (2-by-2 squares), then the next smallest (3-by-3 squares) and then the largest (a 4-by-4 square). There are 16 small squares, 9 squares that are 2-by-2, 4 squares that are 3-by-3, and 1 square that is 4-by-4. That gives a total of 30 squares.

7. **(H)** H is the only letter that matches the attributes. I, N, and Z could all be considered if the student draws them with the middle segment shorter than the other two, but usually this is not the case.

8. **(Accept 1/6, 1/7, or 1/8)** This problem is unusual for students because the piece they are asked to consider is not shown. They will need to know to divide the pie into pieces the same size as the missing piece, and then count all the pieces that would make up the whole pie.

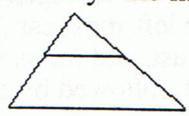
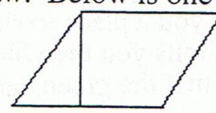
Commentary

Earth, XXII

1. **(72)** This may be solved by repeated addition: 18 + 18 + 18 + 18 = 72. In later years, students will solve the problem by multiplication: 18 centimeters per side times 4 sides gives 72 centimeters.

2. **(Saie has read 1 page more.)** The student needs to first decide how many pages Saie has read. Students might take out a book, and physically go from the top of page 35 to the bottom of page 45. If so, they will count 11 pages. Munjori has only read 10 pages.

3. There are several different ways the line can be draw. Below is one solution for each figure.

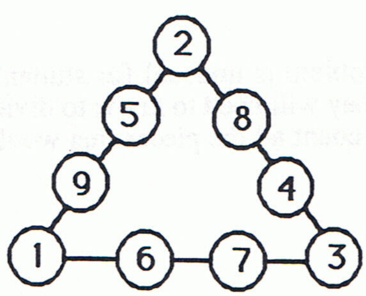



4. **(5 quarters)** His purchases total $1.08. He needs 5 quarters or $1.25 for his purchases.

5. **(1961)** You can turn the number on its head and you see the same number. It will be 6009 before this occurs again.

6. **(left-most spinner)** The probability of spinning a green is 1/2 on the left-hand spinner, and less than that on the other three. The probability of an event in this situation is related to the area of the shape labelled "G." The left-hand figure has the largest area for G. (Note: The third figure may cause concern for some students, as there is no area named G, therefore there is no chance whatsoever of landing on G.)

7. There are many solutions, but they all have in common that the smallest 3 numbers – 1, 2, and 3 – must be in the corners. It's also true that the three highest numbers – 7, 8, and 9 – must all be on different lines. These hints should help students who are having difficulty. One solution is:



8. **(20)** The left-hand balance scale can be used to determine that each apple must weigh 10. Students will find this by guessing what each apple must weigh, and checking to see if 3 apples of that weight total 30. Once they have determined that each apple weighs 10, then the two apples on the right-hand scale must be balanced by a weight of 20.

Commentary

Earth, XXIII

1. **(ruler)** Since 4 pencils balance 2 rulers, 2 pencils must weigh the same as 1 ruler. This means that the ruler is heavier than the pencil.

2. **($6)** Students can easily add $1.25 five times, if they think of this amount as a 1 dollar bill and a quarter. If they put five such amounts together, they have 5 1-dollar bills and 5 quarters, for $5 plus $1 from 4 quarters, plus $0.25 from the left-over quarter. This amounts to $6.25, or a little over $6.

3. **(5)** A pentagon has 5 sides so you would need 5 flags. The purpose of the problem is for students to recognize that geometry words appear in the real world also.

4. **(7)** This is a simple subtraction problem. Some students may want to draw 26 circles for the golf balls she has, then mark out 19, and count the circles left.

5. **(A. 5; B. Pacers; C. See chart below)**

Part (C) above is an intuitive introduction to the concept of an *average*.

6. **($4)** Students will probably put 2 slices together to total $1, and then count by 2s till they get to 8 slices. Some students might add $0.50 eight times.

Commentary

Earth, XXIV

1. **(6)** The students might make an organized list of the ways:

   Oak Street and Main Street
   Oak street and Monroe Street
   Oak Street and Lawn Street
   Fair Street and Main Street
   Fair Street and Monroe Street
   Fair Street and Lawn Street

2. **(A. 389; B. 14; C. 839)**

3. **(No)** Charlie has $4.85 and the gerbil costs $4.86. Charlie is a penny short.

4. **($9\frac{1}{2}$)** This problem may be solved by counting the number of whole units, then putting together half units to make a whole unit, and counting what's left. There are 7 whole units, 2 more whole units by putting together half units, and a half unit left by itself. Some students might know how many there are, but be unfamiliar with writing a mixed number and write it out in words.

5. **(109¢ or $1.09)** Accept either answer.

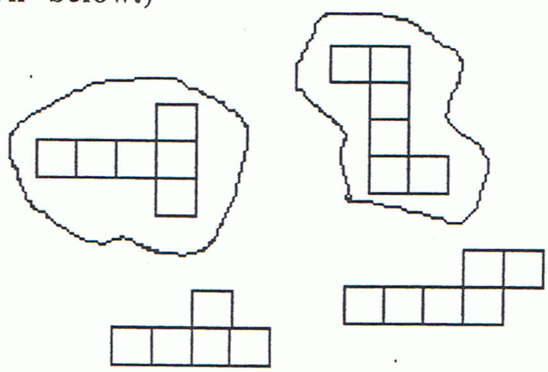
6. **($1.09)** Give the students who use the $ notation in problem 5 credit for this problem also.

7. **(lion: 1, 3; elephant: 3, 4; fish: 4, 2; bird: 5, 3)** The problem introduces the cartesian coordinate system. Students might want to trace the path with their finger, to get to each animal. By convention, the horizontal distance is always given first, followed by the vertical distance.

Commentary
Earth, XXV

1. **(2, balls, bats)** Students will likely add 4 and 6 to get 10 balls, and compare this to 8 bats.

2. **(solutions shown below.)**



3. **(30 minutes)** This multi-step problem is a good activity to be acted out. Students can take 18 tickets and separate them into 6 sets of 3. Then students can find the answer by adding 5 minutes 6 times. $5 + 5 + 5 + 5 + 5 + 5 = 30$ minutes. This type problem will later be solved by multiplication: 6×5 minutes = 30 minutes.

4. **(94)** To find the score of Susie's second game, the student needs to add 20 points to the score of her first game. Then these two scores – 37 and 57 – are added.

5. **(35)** The clock shows 7:25. Students can count by 5s from 7:25 up to 8:00 o'clock, and have 35 minutes. Some students might simply add 5 minutes to the half-hour they see from 7:30 to 8:00, and arrive at 35 minutes more efficiently.

6. **(No)** Students may determine the amount of ribbon needed by counting by 10s: 10, 20, 30, 40, 50, 60, 70, 80, 90. Ninety inches are needed, but a spool only has 86 inches. So the answer is "no."

7. **(Second from left)** The first clue eliminates the 3rd and 4th kites from the left. The second clue eliminates the first kite. The last clue tells you the pattern might be "stripes" since that word rhymes with "yipes." Notice that the first and third clues, taken together, are also enough to determine the kite.

8. **(5)** This problem will later be solved by dividing 31 by 7 and getting 4 r. 3, which tells you that 4 vans is not enough. Therefore 5 are required. At this point, students might solve the problem by drawing 31 stick figures for the students in class, and grouping them in sets of 7 for each van. Three students will be left ungrouped, and need a fifth van.